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The Parameterization Reduction of Soft Sets and its Applications

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Abstract—In this paper, we focus our discussion on the parameterization reduction of soft sets and its applications. First we point out that the results of soft set reductions offered in [1] are incorrect. We also observe that the algorithms used to first compute the reduct-soft-set and then to compute the choice value to select the optimal objects for the decision problems in [1] are not reasonable and we illustrate this with an example. Finally, we propose a reasonable definition of parameterization reduction of soft sets and compare it with the concept of attributes reduction in rough sets theory. By using this new definition of parameterization reduction, we improve the application of a soft set in a decision making problem found in [1].

Keywords—Soft set, Rough set, Fuzzy set, Parameterization reduction, Choice value, Attribute reduction.

1.INTRODUCTION

Many practical problems within fields as diverse as economics, engineering, environment, social science, and medical science involve data that contain uncertainties. These uncertainties cannot be handled using traditional mathematical tools but may be dealt with using a wide range of existing theories such as probability theory, theory of fuzzy sets [2], theory of intuitionistic fuzzy sets [3], theory of vague sets [4], theory of interval mathematics [5], and theory of rough sets [6]. However, as pointed out in [7] that all of these theories have their own difficulties. In [1,7] the authors suggested that one reason for these difficulties may be due to the inadequacy of the theories' parameterization tools. Consequently, Molodtsov posited the concept of soft set as a new mathematical tool for dealing with uncertainties that was free from the difficulties that have troubled the usual theoretical approaches [7]. He pointed out several directions for

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the applications of soft sets. In [1] Maji *et al.* presented an application of soft sets that, in combination with rough sets, addressed a decision-making problem. The problem is represented in the form of an information table and the reduction of the knowledge representation system in rough set theory to define the reduct-soft-set of a soft set is employed. They then proposed an algorithm to select the optimal choice object. Their algorithm uses fewer parameters to select the optimal objects for a decision problem. However, in the decision-making problem in [1], there is a straightforward relationship between the decision values of objects and the conditional parameters, i.e., the decision values is computed with respect to the conditional parameters. This is quite different to the case of rough sets while in rough set theory the decision attributes are not computed according to the conditional attributes. When dealing with their decision-making problem in [1], the authors did not pay more attention to this basic difference between rough sets and soft set. Although their idea of parameterization reduction is meaningful, it may leads to some ineluctable problems. There are two problems in their approach. First, the result of the computing reduction in their example is incorrect. Second, their algorithms to first compute the reduct-soft-set and then to compute the choice value to select optimal objects for decision making is not reasonable as it means that the optimal choice objects could be changed after the reduction of a soft set. The application of weighted soft sets in [1] also suffers from the same two problems. These statements will be analyzed in detail. In this paper, we address all of these problems and then present a new definition of parameterization reduction of soft sets and use this to improve the applications in [1]. We also compare this new definition of parameterization reduction with the concept of attributes reduction in rough set theory. The idea of our new definition of the parameterization reduction of soft set is similar to the idea of attributes reduction found in rough set theory but is applied to different methods.

This paper is organized as follows. Section 2 presents basic definitions of soft and rough sets. Section 3 analyses the material put forward in [1]. In the last section, we propose a new definition of parameterization reduction of soft sets and improve the applications in [1].

2. PRELIMINARIES

In this section, we present the notion of soft sets in [7] and some definitions of rough sets. Let U be an initial universe set and let E be a set of parameters.

DEFINITION 2.1. *(See [7].) A pair (F, E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U.*

In other words, the soft set is a parameterized family of subsets of the set U. Every set $F(\varepsilon)$, $\varepsilon \in E$, from this family may be considered as the set of ε-elements of the soft set (F, E), or as the set of ε-approximate elements of the soft set. As an illustration, some examples such as fuzzy sets and topological spaces were listed in [7]. The way of setting (or describing) any object in soft set theory differs in principle from the way it is used in classical mathematics. In classical mathematics, we construct a mathematical model of an object and define the notion of the exact solution of this model. Usually the mathematical model is too complicated that we cannot find the exact solution. We therefore introduce the notion of approximate solution and calculate that solution. In soft set theory, we have the opposite approach to this problem. The initial description of the object has an approximate nature, and we do not need to introduce the notion of exact solution.

The absence of any restrictions on the approximate description in soft set theory makes it in practice very convenient and easy to apply. We can use any parameterization with the help of words and sentences, real numbers, functions, mappings, and so on.

Assume that we have a binary operation, denoted by $*$, for subsets of the set U. Let (F, A) and (G, B) be soft sets over U. Then, the operation $*$ for soft sets is defined in the following way: $(F, A) * (G, B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) * G(\beta)$, $\alpha \in A, \beta \in B$, and $A \times B$ is the Cartesian product of the sets A and B.

This definition takes into account the individual nature of any soft set.

DEFINITION 2.2. *(See [8].) A knowledge representation system can be formulated as a pair $S = (U, A)$, where U is a nonempty finite set called the universe, and A is a nonempty finite set of primitive attributes.*

Every primitive attribute $a \in A$ is a total function $a : U \to V_a$, where V_a is the set of values of a, called the domain of a.

DEFINITION 2.3. *(See [8].) With every subset of attributes $B \subseteq A$, we associate a binary relation $\mathrm{IND}(B)$, called an indiscernibility relation, defined by*

$$\mathrm{IND}(B) = \{(x, y) \in U \times U : a(x) = a(y),\ \forall a \in B\}.$$

Obviously, $\mathrm{IND}(B)$ is an equivalence relation and $\mathrm{IND}(B) = \bigcap_{a \in B} \mathrm{IND}(a)$.

Suppose $V_a = \{\varepsilon_a^1, \varepsilon_a^2, \dots, \varepsilon_a^{n(a)}\}$. Define $F_a : V_a \to P(U)$ as $F_a(\varepsilon_a^i) = \{x \in U : a(x) = \varepsilon_a^i\}$, then (F_a, V_a) is a soft set. Suppose$A = \{a_1, a_2, \dots, a_m\}$, then $S = (U, A)$ can be expressed as a soft set $(F, V_{a_1} \times V_{a_2} \times \dots \times V_{a_m}) = (F_{a_1}, V_{a_1}) \cap (F_{a_2}, V_{a_2}) \cap \dots \cap (F_{a_m}, V_{a_m})$. For every $(p_1, p_2, \dots, p_m) \in V_{a_1} \times V_{a_2} \times \dots \times V_{a_m}$, $F(p_1, p_2, \dots, p_m) = F_{a_1}(p_1) \cap F_{a_2}(p_2) \cap \dots \cap F_{a_m}(p_m)$. All nonempty sets of $F(p_1, p_2, \dots, p_m)$ form the collection of the equivalence classes of $\mathrm{IND}(A)$. Thus, the soft set can be applied to express a knowledge representation system.

DEFINITION 2.4. *(See [8].) Let R be a family of equivalence relations and let $A \in R$. We say that A is dispensable in R if $\mathrm{IND}(R) = \mathrm{IND}(R - \{A\})$; otherwise A is indispensable in R. The family R is independent if each $A \in R$ is indispensable in R; otherwise R is dependent. $Q \subset P$is a reduction of P if Q is independent and $\mathrm{IND}(Q) = \mathrm{IND}(P)$, that is to say Q is the minimal subset of P that keeps the classification ability. The set of all indispensable relations in P will be called the core of P, and will be denoted as $\mathrm{CORE}(P)$. Clearly, $\mathrm{CORE}(P) = \cap \mathrm{RED}(P)$, where $\mathrm{RED}(P)$ is the family of all reductions of P.*

DEFINITION 2.5. *(See [1].) For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a soft subset of (G, B) if*

(i) $A \subset B$,
(ii) $\forall \varepsilon \in A$, $F(\varepsilon)$, *and* $G(\varepsilon)$ *are identical approximations.*

We write $(F, A) \tilde{\subset} (G, B)$.

3. ANALYSIS OF THE APPLICATION OF SOFT SET IN [1]

In [1], Maji*et al.* presented an application of soft set theory in a decision making problem with the help of rough approach. The problem is described as follows.

Let $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ be a set of six houses, $E = \{$expensive; beautiful; wooden; cheap; in green surroundings; modern; in good repair; in bad repair$\}$, be a set of parameters.

Consider the soft set (F, E) which describes the 'attractiveness of the house', given by

$$\begin{gathered}(F, E) = \{\text{expensive houses} = \phi,\ \text{beautiful houses} = \{h_1, h_2, h_3, h_4, h_5, h_6\}, \\ \text{wooden houses} = \{h_1, h_2, h_6\},\ \text{modern houses} = \{h_1, h_2, h_6\}, \\ \text{houses in bad repair} = \{h_2, h_4, h_5\},\ \text{cheap houses} = \{h_1, h_2, h_3, h_4, h_5, h_6\}, \\ \text{houses in good repair} = \{h_1, h_3, h_6\},\ \text{houses in green surroundings} = \{h_1, h_2, h_3, h_4, h_6\}\}.\end{gathered}$$

Suppose that, Mr. X is interested in buying a house on the basis of his choice parameters 'beautiful', 'wooden', 'cheap', 'in green surroundings', 'in good repair', etc., which constitute the subset $P = \{$beautiful, wooden, cheap, in green surroundings, in good repair$\}$ of the set E. That means, out of available houses in U, he is to select that house which qualifies with all (or with maximum number of) parameters of the soft set P.

Table 1.

U	e_1	e_2	e_3	e_4	e_5
h_1	1	1	1	1	1
h_2	1	1	1	1	0
h_3	1	0	1	1	1
h_4	1	0	1	1	0
h_5	1	0	1	0	0
h_6	1	1	1	1	1

To solve this problem, the soft set (F, P) is firstly expressed as a binary table as shown below. If $h_i \in F(e_j)$ then $h_{ij} = 1$, otherwise $h_{ij} = 0$, where h_{ij} are the entries in Table 1.

Thus, a soft set can now be viewed as a knowledge representation system where the set of attributes is replaced by a set of parameters.

Consider the tabular representation of the soft set (F, P). If Q is a reduction of P, then the soft set (F, Q) is called the reduct-soft-set of the soft set (F, P).

The choice value of an object $h_i \in U$ is c_i, given by $c_i = \sum_j h_{ij}$, where h_{ij} are the entries in the table of the reduct-soft-set.

The algorithm presented in [1] for Mr. X to select the house he wishes is listed as follows.

1. Input the soft set (F, E),
2. Input the set P of choice parameters of Mr. X which is a subset of E,
3. Find all reduct-soft-sets of (F, P),
4. Choose one reduct-soft-set say (F, Q) of (F, P),
5. Find k, for which $c_k = \max c_i$.

Then h_k is the optimal choice object. If k has more than one value, then any one of them could be chosen by Mr. X using his option.

In [1], it is claimed that $\{e_1, e_2, e_4, e_5\}$ and $\{e_2, e_3, e_4, e_5\}$ are two reductions of $P = \{e_1, e_2, e_3, e_4, e_5\}$. But $\{e_1, e_2, e_4, e_5\}$ and $\{e_2, e_3, e_4, e_5\}$ are not really the reductions of $P = \{e_1, e_2, e_3, e_4, e_5\}$. Our following computing results will illustrate this.

Suppose R_P is the indiscernibility relation induced by $P = \{e_1, e_2, e_3, e_4, e_5\}$, then the partition defined by R_P is $\{\{h_1, h_6\}, \{h_2\}, \{h_3\}, \{h_4\}, \{h_5\}\}$. If we delete $\{e_1, e_3\}$ from P, then the indiscernibility relation and the partition are invariant, so both of e_1 and e_3 are dispensable in P by Definition 2.4. If we delete one of $\{e_2, e_4, e_5\}$ from P, then the indiscernibility relation and the partition would be changed, thus all of these three parameters are indispensable. For example, suppose we delete $\{e_2\}$ from P, then the partition is changed to $\{\{h_1, h_3, h_6\}, \{h_2, h_4\}, \{h_5\}\}$. So by Definition 2.4, we know $\{e_2, e_4, e_5\}$ is in fact the reduction of $P = \{e_1, e_2, e_3, e_4, e_5\}$. From table 1 we can also conclude that e_1 and e_3 are not relevant and will not affect the choices of the house since they take the same values for every house.

On the other hand, in this algorithm they compute the reduction of the soft set in Step 3 before computing the choice value in Step 5, which would lead to two problems. First, after reduction, the objects that take max choice value may be changed, so it is possible that the decision after reduction is not the best one. Second, since the reductions of soft set are not unique, it is possible that there would be a difference between the objects that take max choice value obtained using different reductions. In these two cases, the choice object may not be optimal or may be quite difficult to select. Furthermore, even if these two problems do not appear in the example presented in [1], it is highly possible that they appear in other situations. The following example illustrates this.

EXAMPLE 3.1. Suppose we have a soft set (F, E) with the tabular representation displayed in Table 2.

Clearly, $c_2 = 5$ is the biggest choice value, thus h_2 takes the max choice value and will be the optimal choice object. Suppose R_E is the indiscernibility relation induced by E, then the partition

Table 2.

U	e_1	e_2	e_3	e_4	e_5	e_6	e_7
h_1	1	0	1	1	1	0	0
h_2	0	1	1	1	0	1	1
h_3	0	0	1	0	1	0	1
h_4	1	0	1	1	0	0	0
h_5	1	0	1	0	0	1	0
h_6	0	1	1	1	1	0	0

induced by R_E is $\{\{h_1\},\{h_2\},\{h_3\},\{h_4\},\{h_5\},\{h_6\}\}$. The partition obtained from $\{e_1,e_4,e_5\}$ is invariant. If we delete one of $\{e_1,e_4,e_5\}$ then the partition is changed, so $\{e_1,e_4,e_5\}$ is a reduction of (F,E). For example, if we delete e_1 from $\{e_1,e_4,e_5\}$, then the partition will be changed to $\{\{h_1,h_6\},\{h_2,h_4\},\{h_3\},\{h_5\}\}$. Similarly we can examine that $\{e_2,e_4,e_5\}$ is the reduction of (F,E). For $\{e_1,e_4,e_5\}$, h_1 takes the max choice value and will be the optimal choice object with respect to $\{e_1,e_4,e_5\}$, while h_6 takes the max choice value for $\{e_2,e_4,e_5\}$ and will be the optimal choice object with respect to $\{e_2,e_4,e_5\}$. This means that the optimal object is changed after reduction(the optimal choice object is not h_2) and that different reductions decide different optimal objects. If we select the choice objects according to $\{e_1,e_4,e_5\}$ and $\{e_2,e_4,e_5\}$, we will miss the real optimal one. In other words, both of the predicted difficulties do in fact appear.

As Example 3.1 shows that the algorithm presented in [1], which first computes the reduct-soft-set then computes the choice value, is not error-free. For the application found in [1] the choice values of objects are obtained by the number of parameters the object belongs to, thus there is a straightforward relationship between the choice values and the conditional parameters. But for the rough set theory there is no such kind of straightforward relationship between the decision attributes and the conditional attributes, i.e., the decision attributes values are not briefly computed by the conditional attributes values. This statement is the key difference between soft sets and rough sets. In [1], they make the choice values as the decision parameter and try to find minimal subset of conditional parameter set by using reduction in rough set theory to keep the optimal choice object. However, the attributes reduction in rough set theory is designed to find a minimal attributes set that retains the classification ability of the indiscernibility relation. Since choice values in soft set is not decided by the classification ability of the indiscernibility relation, the attributes reduction can not be applied to reduce the number of parameters to keep the optimal choice objects in soft set. Otherwise it is possible that the optimal choice object may be changed after reduction as indicated by Example 3.1. If the parameters set E is divided into two parts, i.e., $E = E_1 \cup E_2$, where E_1 is the conditional parameters set and E_2 is the decision parameters set, here E_2 is not computed by E_1, that is to say there is no straightforward relationship between E_1 and E_2, and either E_1 and E_2 induce indiscernibility relation or partition on the universe, then E_1 and E_2 can be viewed as conditional and decision attributes in rough set theory respectively. We can find the minimal subset of E_1 to keep the classification ability of E_1 relative to E_2 invariant. This is just the concept of relative reduction in rough set theory [8] and is totally different from the decision-making problem in [1]. In [1] the authors did not distinguish between these two cases. One should notice in the soft set as shown in Table 1 that no matter how the parameters are reduced, h_1 and h_6 could be selected as optimal objects. So this application problem is a very special case and the method of introducing reduct-soft-set in [1] is meaningless to deal with this application problem, which could only result in possibly misleading/wrong final decision.

In Section 3.5 of [1], a weighted table of a soft set is presented by having $d_{ij} = w_j \times h_{ij}$ instead of 0 and 1 only, where h_{ij} are the entries in the table of the soft set and w_j are weights of e_j. The weighted choice value of an object $h_i \in U$ is c_i, given by $c_i = \sum_j d_{ij}$. By imposing weights on his choice parameters, Mr. X could now use the following revised algorithm for arriving at his final decisions.

REVISED ALGORITHM FOR SELECTION OF THE HOUSE.

1. Input the soft set (F, E)
2. Input the set P of choice parameters of Mr. X which is a subset of E
3. Find all reduct-soft-sets of (F, P)
4. Choose one reduct-soft-set say (F, Q) of (F, P)
5. Find weighted table of the soft set (F, Q) according to the weights decided by Mr. X
6. Find k, for which $c_k = \max c_i$

Then h_k is the optimal choice object. If k has more than one value, then any one of them could be chosen by Mr. X using his option.

Clearly this revised algorithm still suffers from the same two problems discussed earlier.

This analysis seems to show that soft set theory is quite different from rough set theory and that attributes reduction in rough set theory usually cannot be applied to the decision problems as mentioned in [1]. In Section 4, we introduce the parameterization reduction of soft sets to deal with the decision problems in [1].

4. PARAMETERIZATION REDUCTION OF SOFT SETS

Suppose $U = \{h_1, h_2, \dots, h_n\}$, $E = \{e_1, e_2, \dots, e_m\}$, (F, E) is a soft set with tabular representation. Define $f_E(h_i) = \sum_j h_{ij}$ where h_{ij} are the entries in the table of (F, E). Denote M_E as the collection of objects in U which takes the max value of f_E. For every $A \subset E$, if $M_{E-A} = M_E$, then A is called a dispensable set in E, otherwise A is called an indispensable set in E. Roughly speaking, $A \subset E$ is dispensable means that the difference among all objects according to the parameters in A does not influence the final decision. The parameter set E is called independent if every $A \subset E$ is indispensable in E, otherwise E is dependent. $B \subseteq E$ is called a reduction of E if B is independent and $M_B = M_E$, i.e., B is the minimal subset of E that keeps the optimal choice objects invariant. Clearly, after the reduction of the parameter set E, we have less parameters and the optimal choice objects have not been changed.

The reduction of parameter sets in soft set theory and attributes reduction in rough set theory are in some ways similar to the approach of finding minimal parameters sets or attributes sets in decision-making but they use different methods. In rough set theory, as indicated by Definition 2.4, they define single dispensable attribute while in soft set theory we cannot define a single dispensable parameter as the dispensable attribute. This is because in soft set the decision value $f_E(h_i)$ is computed by the number of parameters that h_i takes the value of 1, the optimal choice objects is obtained by the order of $f_E(h_i)$. Although a single parameter may influence the order of $f_E(h_i)$, it is possible there is another parameter, such that these two parameters do not influence the order of $f_E(h_i)$. That is for a parameter $e \in E$ satisfying $M_{E-\{e\}} \neq M_E$, it is possible that there exists an $e' \in E$, such that $M_{E-\{e,e'\}} = M_E$. For instance, in Example 3.1 $M_E = \{h_1, h_6\}$, $M_{E-\{e_2\}} = \{h_1, h_3, h_6\}$ and $M_{E-\{e_2\}} \neq M_E$, but $M_{E-\{e_1,e_2\}} = \{h_1, h_6\} = M_E$. This case is not shared by rough set theory, i.e., in rough set theory if an attribute is indispensable, any set of attributes containing this attribute will also be indispensable. This means that without this set the ability of the knowledge representation system for solving classification problems will be changed.

However, in rough set theory the attributes reduction is designed to keep the classification ability of conditional attributes relative to the decision attributes. There is not straightforward connection between the conditional attributes and the decision attributes. But for the soft set, the connection between the decision values and the conditional parameters are straightforward, i.e., the decision values are computed by the conditional parameters, and the reduction of parameters is designed to offer minimal subset of the conditional parameters set to keep the optimal choice objects. Now we know that the problems tackled by attributes reduction in rough set theory and parameters reduction in soft set theory are different and their methods are also different, which has been analyzed in previous paragraph. Thus the reduction of parameter sets in soft set theory and the reduction of attributes in rough set theory are different tools for different purposes. In general, one cannot be applied in the place of the other.

For the soft set in Table 1, if we delete e_1, e_3 and e_4 from P, then the optimal choice objects are unchanged. If we delete any subset of P which include at least one of e_2 and e_5, then the optimal choice objects will be changed. For example, if we delete $\{e_2, e_4\}$ from P, then the optimal choice objects will be $\{h_1, h_3, h_6\}$. For $\{e_2, e_5\}$ the optimal choice objects are not changed. Thus the soft set in Table 1 has a parameter reduction $\{e_2, e_5\}$. This means $\{e_2, e_5\}$ are the key parameters in Mr. X's selection of a house. However, $\{e_2, e_5\}$ is not the attributes reduction of P and the attributes reduction $\{e_2, e_4, e_5\}$ is not the parameter reduction of P since it is not the minimal parameter set to maintain the optimal choice objects.

In what follows, we employ our Example 3.1 to illustrate our idea of parameterization reduction. As we mentioned before, for the soft set of Example 3.1, the optimal choice object is h_2. By our definition of parameterization reduction we can examine that $\{e_2, e_6\}$ and $\{e_6, e_7\}$ are two parameterization reductions (not all) of E since they agree to the optimal choice object as h_2, but they are really not the attributes reductions of E since they induce different partitions. As analyzed in previous section, we know $\{e_2, e_4, e_5\}$ is an attributes reduction of E, but it is really not the parameterization reduction of E since it presents another optimal choice object h_6. Thus the analysis of the soft set in Table 1 and Example 3.1 confirm the difference between the parameterization reduction of soft sets and attributes reduction of rough sets.

For the weighted soft sets, we just need to change h_{ij} to $w_j \times h_{ij}$ in the soft set table, then it is possible to propose a similar idea for presenting the reduction of parameter sets for weighted soft sets and this can be applied to improve the decision problem with the weighted soft set in [1].

In a fixed-decision problem where the final decision is unknown, the parameter reduction has only one application, i.e., to present the key parameters. However, if we want to discover knowledge from a data set using a soft set with tabular representation where the decision attribute is given, the parameter reduction can offer optimal parameter sets for newly input/testing objects. This is due to the fact that the complexity of computing the decisions can be reduced by the action of attributes reduction in rough set theory. We shall consider this in an upcoming paper.

5. CONCLUSION

The purpose of this paper is to point out some incorrect and unreasonable statements in [1]. The idea in [1] of employing the attributes reduction in rough set theory to reduce the number of parameters to compute the optimal objects seems meaningless. If their algorithms are applied to other similar decision problems, they will fail. In order to solve these problems, we have proposed a new definition of parameter reduction for soft sets and have used it to improve the application of soft sets to the decision making problem in [1], the basic difference between parameterization reduction of soft sets and attributes reduction in rough sets is also mentioned. The parameter reduction presented in this paper may well play an important role in some knowledge discovery problems.

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